Heavily Enhanced Dynamic Stark Shift in a System of Bose Einstein Condensation of Photons

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The dynamic Stark shift of a high-lying atom in a system of Bose Einstein condensation (BEC) of photons is discussed within the framework of nonrelativistic quantum electrodynamics (QED) theory. It is found that the Stark shift of an atom in BEC of photons is modified by a temperature dependent factor F_{BEC}, compared to that in a normal two-dimensional photonic fluid. In photonic BEC, the value of Stark shift is always greater than that in two-dimensional free space. Physical origin of the phenomenon is presented and potential application is also discussed.

I. INTRODUCTION

Bose Einstein condensation (BEC), predicted by Einstein [1] in 1924, is a remarkable state of matter. For a bosonic system at low temperature, significant friction of particles would spontaneously condensates into zero momentum state with minimum free energy of the system. BEC was first observed in rubidium by Anderson *et al.* [2] in 1995. Thereafter, the phenomenon was revealed in many other systems of atoms or quasi-particles, such as sodium [3], lithium [4], cesium [5], potassium [6], hydrogen [7], polariton [8], *etc.* However, BEC of the simplest boson system, the photon system, was not observed until 2010 [9]. What impeded us so long from realizing Bose Einstein condensate (BEc, lower case "c" to distinguish from BEC) of photons is that in a normal blackbody radiation cavity, the photon is massless and the photon number is not conserved which leads to a vanishing chemical potential. In 2010, Klaers *et al.* [9] have overcome both obstacles by confining laser pumping light in a two-dimensional microcavity which is filled with dye and bounded by two highly reflective concave mirrors. They established the conditions required for the light to thermally equilibrate as a gas of conserved particles rather than as an ordinary blackbody radiation.

As is well known, quantum optical effects of atoms are not only dependent on their internal structure, but also on external electromagnetic environment. People have explored various systems in modified electromagnetic environment such as in dielectric medium[10, 11], photonic crystals[12, 13], and optical microwave guides[14]. The results show that the energy-level shift is modified accordingly. For example, Wang *et al.*[15] predicted that the dominant contribution to the Lamb shift comes from emission of real photon in photonic crystals that the Lamb shift can be enhanced by 1 or 2 orders of magnitude, termed as 'giant' Lamb shift. In a recent work, we investigated the Lamb shift of a hydrogen atom inside a Kerr nonlinear body (KNB) and also found that the modification of the Lamb shift is a 'giant' one [16]. What is more, we found the dynamic Stark shift in KNB revealed a temperature dependency [17]. Inspired by the experimental demonstration of BEC of photons, in this paper, we aim to investigate the dynamic Stark shift of an atom in a system of BEC of photons. We find that in such a system, the dynamic Stark shift of a high-lying atom can be heavily enhanced.

The remainder of this paper is organized as follows. In Sec. II, we configure the microcavity for BEc of photon system and theoretically model the BEC of weakly interacting photons in a two-dimensional optical cavity. The dynamic Stark shift of a high-lying atom in photonic BEc is derived in Sec. III. In Sec. IV, we discuss the order of modified shift and the role of dimension. Finally, we make a brief conclusion in Sec. V.

II. BOSE EINSTEIN CONDENSATION OF PHOTON SYSTEM

In this section, we theoretically establish the model for BEC of a two-dimensional photon gas. We first base the design on Klaers' work to configure the two-dimensional microcavity in which photonic BEC may realize. Then we turn to reexamine the dispersion relations of two-dimensional free and weakly-interacting photon gas. Finally, we discuss the conditions of BEC of photons.

A. Microcavity resonator

The first difficulty stands in our way is that the chemical potential of photon system is vanishing in normal blackbody radiation, that is, the photon number is not conserved when the temperature of the photon gas is varied. However, a prerequisite of BEC is that the chemical potential be non-vanishing [18]. For a black body radiation, photons disappear in the cavity walls instead of occupying the cavity ground state [9]. Luckily, number conserving thermalization of two-dimensional photon gas was experimentally achieved by Klaers *et al.* [9], theoretical work was previously done by Chiao *et al.* [19].

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Typically, a microcavity resonator is required to be com-

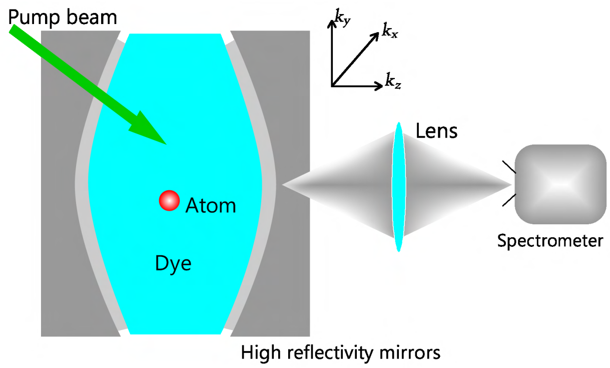

FIG. 1. Schematic of the optical microcavity. Consisting of two high-reflectivity curved mirrors which are fulfilled with dye solution. An atom is immersed inside the microcavity away from the laser path.

posed of highly reflective mirrors with reflectivity over 99.997% at both end, acting as the walls, as shown in FIG. 1. Separation between surfaces of adopted curved mirrors are described by equation $D(r) = D_0 - 2(R - \sqrt{R^2 - r^2})$ with r being the distance from optical axis. A laser of 532 nm pumps dye onto the cavity at an angel of 45° to optical axis. The atom should be located inside of the cavity and completely immersed in solution but away from pumping beam to avoid possible pumping light excitation. Generally, the atoms couples with photons forming polaritons, a bosonic quasi-particle. However, due to the frequent collisions of photons and dye solution, this coupling is broken[20, 21]. Thus we may convince ourselves that the BEC we observed here is real BEc of photons instead of that of polaritons.

B. Free Photon Dispersion Relation

The mirrors have confined the photons to a two-dimensional plane, simultaneously imposing a boundary condition on both sides of inside electric field. The allowed z-component which is along the optical axis of wave vector of photon is $k_z(r) = n'\pi/D(r)$. n' is thus an integer. The electric field vanishes at the reflecting surface thus imposing a quantization condition for a component of photon wave vector which denotes as z. Photons are trapped inside the resonator. In this two-dimensional plane, a plane-mode wave propagates at small angle with respect to z axis, the effective mass of a non-relativistic particle is $m_\gamma = \hbar\omega/c^2$ [22].
For a two-dimensional noninteracting photon inside the cavity resonator, the de Broglie dispersion relation or energy-momentum relation under small-angel approximation is

$$\epsilon(\boldsymbol{p}) = m_\gamma c^2 + \frac{\boldsymbol{p}^2}{2m_\gamma}. \tag{1}$$

We note that in present cavity resonator k_z, which is greater than that of x or y component k_x, k_y under the small angle propagation. Thus, we ignore the contribution of k_z, giving

$$\boldsymbol{p}^2 = p_x^2 + p_y^2.$$

Confining our sight to the two-dimensional plane, we may view free photons as normal particles in a transverse resonant plane. This may owe to the fact that number conservation is achieved by scattering off process of photons and dye molecules, which make it a canonical ensemble.

C. Bogoliubov Dispersion Relation for Photon Gas

There exists Bose Einstein condensation in an ideal Bose system at absolute zero temperature. This feature survives in the case of the weakly interacting Bose system, since as the interaction vanishes, one should recover the Bose Einstein condensate state[23]. From the work by Klaers *et al.*[9], we may find the magnitude of the interaction energy of photon knowing the intensity of light $I(r)$ and relative refraction index μ

$$E_{\text{int}} = m_\gamma c^2 \mu I(r). \tag{2}$$

We rewrite it using what applied in Gross−Pitaevskii method, introducing a dimensionless parameter [24] $g = -m_\gamma^4 c^6/2\pi h^3 nq$ to describe the strength of interaction. Here, c is the speed of light, n is the the quantum concentration of current system, and q is a constant. So we rewrite the interaction energy E_{int} in term of g

$$E_{\text{int}} = \hbar^2/m_\gamma g N_0 n^2.$$

In this cavity, $g \sim (7 \pm 3) \times 10^{-4}$, far below that reported in two-dimensional quantum gas experiment, in order of $-2 \sim -1$. This indicates that the photons are weakly interacting which enable us to apply Bogoliubov transform in such a two-dimensional quantum gas system.
Since this system is open, i.e., it is connected to an external reservoir of particles, so that the average of total particle number fluctuate around an average value, then the total particle number needs only be conserved on average. To formally illustrate it in the expression of Hamiltonian, we introduce Lagrange multiplier μN_{open}, like that done in statistical mechanics.

$$H = H' - \mu N_{\text{open}} \tag{3}$$

where $N_{\text{open}} = \sum_{\boldsymbol{p}} a_{\boldsymbol{p}}^\dagger a_{\boldsymbol{p}}$ is total number operator and μ the chemical potential. Experimentally, pump beam in the configuration is the physical implementation of external reservoir. The overall energy is thus given by the equations above. Now, we study the Hamiltonian of the system in details

$$H' = H_{\text{int}} + H_{\text{non-int}}. \tag{4}$$

For a free boson system, with de Broglie dispersion relation in Eq. (1), the Hamiltonian is

$$H_{\text{non-int}} = \sum_{\boldsymbol{p}} \epsilon(\boldsymbol{p}) a_{\boldsymbol{p}}^{\dagger} a_{\boldsymbol{p}}. \tag{5}$$

We introduce the creation and annihilation operators $a_{\boldsymbol{p}}^{\dagger}$ and $a_{\boldsymbol{p}}$ for photons with momentum $\boldsymbol{p}$, respectively. We know that in a weakly-interacting Bose gas at absolute zero temperature, most particles with zero momentum occupying condensate state, are in great number N_0. For such ground state $|N_0\rangle$, the zero-momentum operator $a_0^{\dagger}$ and a_0 obey the following relations

$$a_0 |N_0\rangle = \sqrt{N_0}\, |N_0\rangle \,,$$

$$a_0^{\dagger} |N_0\rangle = \sqrt{N_0+1}\, |N_0\rangle \,.$$

Noting that N_0 is large enough in this case, so, we may approximately take $\sqrt{N_0+1} \approx \sqrt{N_0}$. The operators also satisfy Bose commutation relations

$$\begin{aligned} \left[a_{\boldsymbol{p}}, a_{\boldsymbol{q}}^{\dagger}\right] &= \delta_{\boldsymbol{p},\boldsymbol{q}}, \\ \left[a_{\boldsymbol{p}}, a_{\boldsymbol{q}}\right] &= \left[a_{\boldsymbol{p}}^{\dagger}, a_{\boldsymbol{q}}^{\dagger}\right] = 0. \end{aligned} \tag{6}$$

The interaction term describes the momentum transfer of two photons arising from potential energy $V(\boldsymbol{p})$, it represents annihilation of two photons with momentum $\boldsymbol{p}$ and $\boldsymbol{q}$, with the creation of two photons with momentum $\boldsymbol{p}+\boldsymbol{\kappa}$ and $\boldsymbol{q}-\boldsymbol{\kappa}$. Specifically, the annihilation of two zero momentum photons giving rise to non-zero ones with momentum transfer $\boldsymbol{\kappa}$ within this pair is the phase transition from condensate state to normal state. The interaction is reasonably in the following form

$$H_{\text{int}} = \sum_{\boldsymbol{p},\boldsymbol{q}} \frac{1}{2} V(\boldsymbol{\kappa})\, a_{\boldsymbol{p}+\boldsymbol{\kappa}}^{\dagger} a_{\boldsymbol{q}-\boldsymbol{\kappa}}^{\dagger} a_{\boldsymbol{p}} a_{\boldsymbol{q}}. \tag{7}$$

In BEC, since only small momenta are concerned, we consider the zero-momentum term of interaction potential

$$V_0 = \frac{4\pi\hbar^2 s}{m_\gamma} \tag{8}$$

where $s = g/2\pi$ is s-wave scattering length and m_γ is effective mass of photons[18]. And we assume that $V(\boldsymbol{p}) = V(-\boldsymbol{p})$ is valid.

Since in BEC, particles are macroscopically occupying the ground state, N_0 is relatively large, that is $N \approx N_0 \gg 1$. Terms with $a_0^{\dagger} a_0 = N_0$ dominates. Under weakly interaction approximation, for interaction terms in (7), only that with N_0 and N_0^2 are preserved, so the Hamiltonian H is

$$\begin{aligned} H \approx\ & \epsilon'(0) + \sum_{\boldsymbol{p}\neq 0} \epsilon'(\boldsymbol{p}) a_{\boldsymbol{p}}^{\dagger} a_{-\boldsymbol{p}} \\ & + \sum_{\boldsymbol{p}\neq 0} N_0 V_{\boldsymbol{p}} (a_{\boldsymbol{p}}^{\dagger} a_{-\boldsymbol{p}}^{\dagger} + a_{\boldsymbol{p}} a_{-\boldsymbol{p}}) \end{aligned} \tag{9}$$

where

$$\epsilon'(0) = N_0 \epsilon(0) - \frac{1}{2} V_0 N_0^2 \tag{10}$$

and

$$\epsilon'(\boldsymbol{p}) = \epsilon(\boldsymbol{p}) + N_0 V_{\boldsymbol{p}} + N_0 V_0 - \mu \tag{11}$$

are modified photon energy.

Following Bogoliubov[25], we now diagonalize the Hamiltonian, that is, transforming the photon operators into quasi-particle ones. Since the Bose commutation relation preserve in such case, transform can be introduced and applied in the following form

$$\begin{aligned} b_{\boldsymbol{p}} &= u_{\boldsymbol{p}} a_{\boldsymbol{p}} + v_{\boldsymbol{p}} a_{-\boldsymbol{p}}^{\dagger}, \\ b_{\boldsymbol{p}}^{\dagger} &= u_{\boldsymbol{p}} a_{\boldsymbol{p}}^{\dagger} + v_{\boldsymbol{p}} a_{-\boldsymbol{p}} \end{aligned} \tag{12}$$

where

$$u_{\boldsymbol{p}}^2 - v_{\boldsymbol{p}}^2 = 1,$$

and

$$\begin{aligned} \left[b_{\boldsymbol{p}}, b_{\boldsymbol{q}}^{\dagger}\right] &= \delta_{\boldsymbol{p},\boldsymbol{q}}, \\ \left[b_{\boldsymbol{p}}, b_{\boldsymbol{q}}\right] &= \left[b_{\boldsymbol{p}}^{\dagger}, b_{\boldsymbol{q}}^{\dagger}\right] = 0. \end{aligned} \tag{13}$$

A diagonal form of weak interaction term of $\boldsymbol{p}$ and $-\boldsymbol{p}$ momenta photons are expected to be

$$H = \sum_{\boldsymbol{p}} \tilde{\epsilon}_{\boldsymbol{p}} \left(b_{\boldsymbol{p}} b_{\boldsymbol{p}}^{\dagger} + \frac{1}{2} \right) + E_0 \tag{14}$$

where E_0 is ground-state energy. Eq. (14) shows that the original system can be reexpressed in term of new set of operators $b_{\boldsymbol{p}}$ and $b_{\boldsymbol{q}}^{\dagger}$ which respectively represents the annihilation and creation of a quasi-particle with energy $\tilde{\epsilon}(\boldsymbol{p})$. Their motion satisfy the following equation in Heisenberg picture

$$i\hbar \frac{\mathrm{d} b_{\boldsymbol{p}}}{\mathrm{d}t} = [b_{\boldsymbol{p}}, H] = \tilde{\epsilon}(\boldsymbol{p}) b_{\boldsymbol{p}}. \tag{15}$$

Substituting Eqs. (12) and (13) into original Hamiltonian (Eq. (9)) and comparing with Eq. (14), we get the following diagonalization conditions

$$\begin{aligned} u_{\boldsymbol{p}} v_{\boldsymbol{p}} &= \frac{1}{2} N_0 V_{\boldsymbol{p}} / \tilde{\epsilon}(\boldsymbol{p}), \\ u_{\boldsymbol{p}}^2 &= \frac{1}{2} [\epsilon'(\boldsymbol{p}) / \tilde{\epsilon}(\boldsymbol{p}) + 1], \\ v_{\boldsymbol{p}}^2 &= \frac{1}{2} [\epsilon'(\boldsymbol{p}) / \tilde{\epsilon}(\boldsymbol{p}) - 1]. \end{aligned} \tag{16}$$

Solving Eq. (11) and Eq. (15) for $\tilde{\epsilon}(\boldsymbol{p})$, we have

$$\begin{aligned} \tilde{\epsilon}(\boldsymbol{p})^2 &= \epsilon'(\boldsymbol{p})^2 - N_0^2 V_0^2 \\ &= \epsilon(\boldsymbol{p})^2 + 2\epsilon(\boldsymbol{p}) N_0 V_0, \end{aligned} \tag{17}$$

so

$$\tilde{\epsilon}(\boldsymbol{p}) = \sqrt{\frac{\boldsymbol{p}^2 N_0 V_0}{m_\gamma} + \frac{\boldsymbol{p}^4}{4m_\gamma^2}}. \tag{18}$$

This is the Bogoliubov dispersion relation for the present system. Specifically, at critical temperature, the two terms in Eq. (18) are equal, so that critical momentum $\boldsymbol{p}_c = 2\sqrt{m_\gamma N_0 V_{\boldsymbol{p}_c}}$ [23]. At low temperature, we adopt long-wavelength approximation so that

$$\tilde{\epsilon}_{\boldsymbol{p}} = \tilde{c}\,|\boldsymbol{p}| \tag{19}$$

where $\tilde{c} = \sqrt{N_0 V_0 / m_\gamma}$ is the sound velocity[18]. Actually, the fluctuation of photon gas described by $\tilde{c}$ and $b_{\boldsymbol{p}}\ b^\dagger_{\boldsymbol{p}}$ suggest that the excitations here is phonon which can be reexpressed as

$$\tilde{c} = \sqrt{\frac{V_0}{m_\gamma}}\sqrt{N - \alpha T}$$

where N is the total number of photons in cavity and $\alpha = m_\gamma k_{\mathrm{B}} \sum_{\boldsymbol{p}\neq 0} \boldsymbol{p}^{-2}$ is a constant independent of $\boldsymbol{p}$ nor temperature T, but concerns the distribution of momenta [18].

D. Condensation Condition

We now examine the condition of BEC. All phase transition has a critical point, here is the critical temperature T_c. Only at low temperature, particles are occupying ground state. Above T_c, the condensate phase vanish, photons are free. For a normal three-dimensional BEc, the critical temperature is [26]

$$T_c = \frac{3.31\hbar^2}{\tilde{g}^{\frac{2}{3}} m_\gamma k_{\mathrm{B}}} \frac{N}{\mathcal{V}}^{\frac{2}{3}} \tag{20}$$

where $\mathcal{V}$ is the volume of the resonator. However, in two-dimensional harmonic tarp, a revising factor $\sqrt{\frac{6}{\pi^2}} \simeq 0.78$ is applied with hard core calculations, see Eq. (18) of Ref. [27]. For photons, the degeneracy $\tilde{g} = 2$. Calculation shows that in present configuration, the critical temperature is about 578.062 K (for data see the caption of FIG.2). That means experiment can be achieved even in room-temperature. However, there exists another condition that quantum concentration constrains

$$n > n_Q = \quad (\frac{m_\gamma k_{\mathrm{B}} T}{2\pi\hbar^2})^{3/2} \tag{21}$$

where n is the quantum concentration of current system, we find it using

$$n = \quad N_0 (\frac{2\pi\hbar^2}{m_\gamma k_{\mathrm{B}} T})^{-3/2}. \tag{22}$$

In present configuration, $n_Q = 1.00828 \times 10^{15}$ and $n = 7.76374 \times 10^{18}$. Condition is satisfied.
Here, the dye solution serves as the heat bath and equilibrates the transverse modal degree of freedom of photons and dye molecules via absorption and re-emission processes. At room temperature, photon frequency is above the low-frequency cut-off, in contrast to the case of a blackbody radiator, for which the photon number is determined by Stefan-Boltzmann law thus reveals a temperature dependency.

III. DYNAMIC STARK SHIFT

Now, we turn to the electromagnetic environment of the atom inside the cavity. First, we write down the Hamiltonian of the system

$$H_{\mathrm{sys}} = H_{\mathrm{free}} + H + H'_{\mathrm{int}} \tag{23}$$

where the free Hamiltonian H_{free} is that of a bare atom and H'_{int} is the interaction Hamiltonian between atom and electric field. This atom can either be at high-lying (Rydberg) state or low-lying state. Typically, we consider an atom with only one electron on its outermost electron shell. Atoms like hydrogen and alkali like rubidium, potassium are widely used in experiment. Much work has already been done by Hollberg *et al.* [28] and Zimmerman *et al.* [29] both theoretically and experimentally. For such an atom, we express the Hamiltonian in form of

$$H_{\mathrm{free}} \left|n\right\rangle = E_n \left|n\right\rangle. \tag{24}$$

The other term H has been derived in the previous section, see Eq. (14). Our interest now is focused on the H'_{int} term, the interaction Hamiltonian between atom and photon fluid is

$$H'_{\mathrm{int}} = -e\boldsymbol{r} \cdot \boldsymbol{E} \tag{25}$$

where $\boldsymbol{r}$ is the radius vector of the electron, $\boldsymbol{E}$ is the electric field induced by BEc of photons, and e is the absolute value of the electron charge. To get the expression of H'_{int}, we need the expression of $\boldsymbol{E}$. Generally, a vector potential of electromagnetic field is

$$\boldsymbol{A} = \sum_{\boldsymbol{p}} \sqrt{\frac{\hbar}{2\mathcal{V}\varepsilon_0 \omega_{\boldsymbol{p}}}} \left(a_{\boldsymbol{p}}(t) + a^\dagger_{\boldsymbol{p}}(t)\right) \hat{e}_{\boldsymbol{p}}. \tag{26}$$

It is worthy of noting that to emphasize the time related term $e^{-i\omega_{\boldsymbol{p}} t}$, we add "$(t)$" to indicate that operators $a_{\boldsymbol{p}}$ is actually time-dependent. For $b_{\boldsymbol{p}}$, it is $e^{-i\tilde{\omega}_{\boldsymbol{p}} t}$ term. $\tilde{\omega}_{\boldsymbol{p}}$ is the angular frequency of phonon with momentum $\boldsymbol{p}$. We have shown that the interaction between the field induced by weakly interacting photons in BEc and atoms can be viewed as the collection of non-interacting excitations that perturb the atoms. Using Bogoliubov transform (Eq. (12)), we rearrange the photon BEc induced

electric field into excitation operators described form. Using equation (12) and (13), the vector potential is

$$\boldsymbol{A} = \sum_{\boldsymbol{p}} \sqrt{\frac{\hbar}{2\mathcal{V}\varepsilon_0\omega_{\boldsymbol{p}}}} e^{\beta} \left(b_{\boldsymbol{p}}(t) + b^{\dagger}_{\boldsymbol{p}}(t)\right) \hat{e}_{\boldsymbol{p}} \tag{27}$$

where

$$e^{\beta} = u_{\boldsymbol{p}} + v_{\boldsymbol{p}} = \sqrt[4]{1 + \frac{N_0 V_{\boldsymbol{p}}}{\epsilon_{\boldsymbol{p}}}}.$$

At critical momentum $\boldsymbol{p}_c$, the interaction potential has a harmonic form[23]

$$V_{\boldsymbol{p}} = \lambda \boldsymbol{p}^2 \tag{28}$$

where $\lambda = 1/4m_{\gamma}N_0$ is a constant with a unit of mass^{-1}. A reasonable assumption is adopted that λ is independent of $\boldsymbol{p}$ under current near zero temperature approximation. With this we evaluate the value of e^{β}

$$\begin{aligned} e^{2\beta} &= \sqrt{1 + \frac{N_0 V_{\boldsymbol{p}}}{\epsilon_{\boldsymbol{p}}}} \\ &= \sqrt{1 + \frac{N_0 \lambda \boldsymbol{p}^2}{\boldsymbol{p}^2/2m_{\gamma}}}. \\ &= \sqrt{1 + 2m_{\gamma}N_0\lambda} \\ &= \sqrt{3/2}. \end{aligned} \tag{29}$$

Now, we turn to the electric field $\boldsymbol{E}$. By using Maxwell equation $\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t}$, the electric field is

$$\boldsymbol{E} = i \sum_{\boldsymbol{p}} \sqrt{\frac{\hbar}{2\mathcal{V}\varepsilon_0\omega_{\boldsymbol{p}}}} e^{\beta} \tilde{\omega}_{\boldsymbol{p}} \left(b_{\boldsymbol{p}}(t) - b^{\dagger}_{\boldsymbol{p}}(t)\right) \hat{e}_{\boldsymbol{p}}. \tag{30}$$

So, we have interaction Hamiltonian

$$H'_{\text{int}} = -ie \sum_{\boldsymbol{p}} \sqrt{\frac{\hbar}{2\mathcal{V}\varepsilon_0\omega_{\boldsymbol{p}}}} e^{\beta} \tilde{\omega}_{\boldsymbol{p}} (b_{\boldsymbol{p}}(t) - b^{\dagger}_{\boldsymbol{p}}(t)) \boldsymbol{r} \cdot \hat{e}_{\boldsymbol{p}}. \tag{31}$$

Now, we consider the perturbation term who bares its first order term being zero due to symmetry of electrical dipole. So we turn to the second order

$$\begin{aligned} \Delta E(\phi) = &\sum_{n',I,\boldsymbol{p}} \frac{\left|\left\langle \phi, \psi^n_{\boldsymbol{p}} \left| H'_{int} \right| I, \psi^{n'}_{\boldsymbol{p}} \right\rangle\right|^2}{E^{\phi}_{\psi^n_{\boldsymbol{p}}} - E^{I}_{\psi^{n'}_{\boldsymbol{p}}}} \\ = &\sum_{I,\boldsymbol{p}} \Big(\frac{\left|\left\langle \phi, \psi^n_{\boldsymbol{p}} \left| H'_{int} \right| I, \psi^{n-1}_{\boldsymbol{p}} \right\rangle\right|^2 \bar{N}_{\boldsymbol{p}}}{E^{\phi} + N_{\boldsymbol{p}}\epsilon'_{\boldsymbol{p}} - \left(E^I + (N_{\boldsymbol{p}} - 1)\epsilon'_{\boldsymbol{p}}\right)} + \\ &\frac{\left|\left\langle \phi, \psi^n_{\boldsymbol{p}} \left| H'_{int} \right| I, \psi^{n+1}_{\boldsymbol{p}} \right\rangle\right|^2}{E^{\phi} + N_{\boldsymbol{p}}\epsilon'_{\boldsymbol{p}} - \left(E^I + (N_{\boldsymbol{p}} + 1)\epsilon'_{\boldsymbol{p}}\right)} \Big) \\ = &\sum_{I,\boldsymbol{p}} \sqrt{\frac{3}{2}} \frac{\hbar}{2\mathcal{V}\varepsilon_0\omega_{\boldsymbol{p}}} \tilde{\omega}^2_{\boldsymbol{p}} \Big(\frac{\left|\left\langle \phi \left| \boldsymbol{r} \cdot \hat{e}_{\boldsymbol{p}} \right| I \right\rangle\right|^2}{E^{\phi} - E^I \pm \epsilon'_{\boldsymbol{p}}} \bar{N}_p \\ &+ \frac{\left|\left\langle \phi \left| \boldsymbol{r} \cdot \hat{e}_{\boldsymbol{p}} \right| I \right\rangle\right|^2}{E^{\phi} - E^I - \epsilon'_{\boldsymbol{p}}} \Big) \\ = &\Delta E^{\text{AC}}_{\phi} + \Delta E^{\text{Lamb}}_{\phi}. \end{aligned} \tag{32}$$

It is worthy of noting that in result Eq. (32), two terms are presented. The first one is the dynamic (or AC) Stark shift induced by real photons, and respectively the second one is the Lamb shift induced by virtual photons or , say, by vacuum. Mass renormalization [30] should be preformed to each term followed by integral ranging from 0 to U. Since we are considering a non-relativistic case, an upper limit of energy, here U, should be applied that is large enough with respect to the energy difference $E^{\phi} - E^{I}$ [31]. Thus we have

$$\begin{aligned} \Delta E^{\text{AC}} = &\sqrt{\frac{3}{2}} \frac{e^2 K c}{4\pi\varepsilon_0 \tilde{c}} (\frac{k_{\text{B}}T}{\hbar c})^2 \sum_I \text{P} \int \text{d}t \left|\boldsymbol{r}_{\phi,I}\right|^2 \frac{t^2}{e^t - 1} \times \\ &\Big(\frac{1}{(E_{\phi} - E_I)/k_{\text{B}}T + t} + \frac{1}{(E_{\phi} - E_I)/k_{\text{B}}T - t} \Big) \end{aligned} \tag{33}$$

where $t = p\tilde{c}/k_{\text{B}}T$, $\left|\boldsymbol{r}_{\phi,I}\right|^2 = \left|\left\langle \phi \left| \boldsymbol{r} \right| I \right\rangle\right|^2$, $K = \int k_z(r)\text{d}r$ and

$$\begin{aligned} \Delta E^{\text{Lamb}} = &\sqrt{\frac{3}{2}} \frac{e^2 K c}{4\pi\varepsilon_0 \tilde{c}} (\frac{k_{\text{B}}T}{\hbar c})^2 \sum_I \text{P} \int \text{d}t \times \\ &\left|\boldsymbol{r}_{\phi,I}\right|^2 \frac{t^2}{(E_{\phi} - E_I)/k_{\text{B}}T - t}. \end{aligned} \tag{34}$$

"P" in the above equations denote that a principle value integral should be preformed instead of the normal one since it is a improper integral.

By using the normal vector potential (Eq.(26)) and its corresponding electric field, we can easily get the dynamic Stark shift in a normal two-dimensional photon gas

$$\begin{aligned} \Delta E^{\text{AC}}_{\text{Normal}} = &\frac{e^2 K}{4\pi\varepsilon_0} (\frac{k_{\text{B}}T}{\hbar c})^2 \sum_I \text{P} \int \text{d}t \left|\boldsymbol{r}_{\phi,I}\right|^2 \frac{t^2}{e^t - 1} \times \\ &\Big(\frac{1}{(E_{\phi} - E_I)/k_{\text{B}}T + t} + \frac{1}{(E_{\phi} - E_I)/k_{\text{B}}T - t} \Big). \end{aligned} \tag{35}$$

Comparing Eq. (33) with Eq. (35), we can rearrange the shift in the form of

$$\Delta E^{\text{AC}} = F_{\text{BEC}} \cdot \Delta E^{\text{AC}}_{\text{Normal}} \tag{36}$$

where

$$F_{\text{BEC}} = \sqrt{\frac{3}{2}} \cdot \frac{c}{\tilde{c}} \tag{37}$$

is the contribution factor of BEc. In this equation, we may read and find what BEc-related shift is different from a normal one. The factor F_{BEC} described the modification with respect to the Stark shift in a normal photon fluid $\Delta E^{\text{AC}}_{\text{Normal}}$.

IV. RESULT AND DISCUSSIONS

From Eq. (36), we can see that the Stark shift in a system of BEC of photons is modified by a factor F_{BEC} com-

pared to that in a normal two-dimensional fluid. To better illustrate the physical interpretation, we can rewrite it in the following form

$$F_{\mathrm{BEC}} = \sqrt{\frac{3m_\gamma}{2V_0}}\frac{c}{\sqrt{N-\alpha T}}.$$

In order to give a numerical impression of F_{BEC}, we take the experimental data from Klaers *et al.* [9]. The effective mass $m_\gamma \approx 6.7\times10^{-36}$ kg, the zero momentum interaction potential $V_0 \approx 3.2\times10^{-33}$ J, the speed of light $c\approx 3\times10^8$ m/s, and the total photon number N no less than 77000. As a result, F_{BEC} is a number of order $4\sim5$. FIG. 2 shows the function of F_{BEC} to αT.

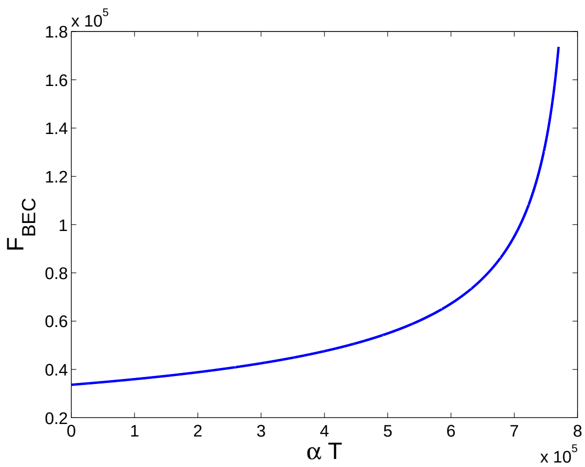

FIG. 2. Modification factor F_{BEC} as a function of modified temperature αT. Data are based on the experiment performed by Klaers *et al.*[9]. $V_0 = 4\pi\hbar^2 s/m_\gamma \approx 3.2\times10^{-33}$ J, cut-off frequency $\omega_{\text{cut-off}}\approx 3\times10^{15}$ Hz, effective mass of photon $m_\gamma \approx 6.7\times10^{-36}$ kg, dimensionless parameter for interaction strength $g\approx10^{-3}$, and the volume $\mathcal{V} = 1.45\times10^{-12}$ m^3.

Near absolute zero temperature, it does not vanish but approximate 6.6315×10^4. In spite that the normal shift is small, the total effect is significant. So, as long as the fluid is in BEC state, the shift is far greater than the normal one.

Up to now, one may raise the question why the dynamic Stark shift is so large in BEC of photons. This can be answered as follows. As is known, the dynamic Stark shift is caused by perturbation of electric field, here, contributed by real photons. The key physical function concerning the atomic QED is the density of state (DOS) of the photonic system. In a non-absorbing linear medium, the DOS is given by

$$\rho(\omega_{\boldsymbol{p}}) = \mathcal{V}\omega_{\boldsymbol{p}}^2/\pi^2c^3. \tag{38}$$

However, in present environment, the photons are replaced by quasi-particles. The DOS has to accordingly change to a quasi-particle described form

$$\rho(\tilde\omega_{\boldsymbol{p}}) = \mathcal{V}\tilde\omega_{\boldsymbol{p}}^2/\pi^2\tilde c^3. \tag{39}$$

We can see that DOS in BEC of photons is a function of the quasi-particle velocity $\tilde c$, thus a temperature dependent value. At near zero temperature, $\tilde c$ is much smaller than c, then the DOS now is relatively much larger than normal one. Much concentrated quasi-particles results in a heavily enhanced Stark shift.

Additionally, we consider the role of dimension of the electromagnetic environment on the dynamic Stark shift. Unlike other photon systems, a two-dimensional system applied here, current resonator have a non-vanishing chemical potential, significantly different from three-dimensional photon system typically encountered in Planck problem. This may be accounted by the repulsive pairwise interactions between photons in a Bose Einstein condensate. To find the chemical potential μ, use

$$\mu = \frac{\partial E_0}{\partial N} \tag{40}$$

where E_0 is ground-state energy $\langle\psi|H|\psi\rangle$. Since it is in superfluidity, which is necessarily Bose-Einstein condensate state, the zero-momentum photons or pairs dominates, thus the other terms are dropped out. Previously, we have $\epsilon_0 = \frac{1}{2}N^2V_0$ that makes

$$\mu = N_0V_0 \tag{41}$$

which clearly shows that a non-vanishing chemical potential is actual in current resonator. The existence of low-dimensional BEC has already been discussed. Back to 1997, Mullin theoretically showed that the Bose gas has a phase transition at some critical temperature T_c in a two-dimensional harmonic trap in the thermodynamic limit [27]. Further, he pointed out that for dimension no less than 1, T_c follows the same expression, however, in one-dimensional case, there is no condensation.

The dynamic Stark shift in a three-dimensional normal fluid given by Farley *et al.* [32]

$$\begin{aligned}\frac{e^2}{6\pi^2\varepsilon_0}(\frac{k_BT}{\hbar c})^3\sum_I \mathrm{P}\int\frac{t^3}{e^t-1}(&\frac{1}{(E_\phi-E_I)/k_BT+t}\\ &+\frac{1}{(E_\phi-E_I)/k_BT-t}).\end{aligned} \tag{42}$$

Compare above equation with that for a two-dimensional case Eq. (35), we may conclude that for Stark shift in system of different dimensions, the dimensionality concerns the exponential of temperature term $k_BT/\hbar c$ and t in integral term. A higher order of dimensions makes the shift far greater in a relatively high temperature. However, in a system of BEC of photons, it can be neglected. In rencent years, the technology of atom manipulation by laser has been developing rapidly. Stark shift is considered to be suitable to compensate the Doppler shift of traveling atom. In a normal electromagnetic environment, the Stark shift is too small to be qualified. However, in the BEC of photons, the heavily enhanced Stark shift seems promising. Due to its large value, a wide range of frequency in Doppler compensation might

be possible. What is more, the heavily enhanced Stark shift might be used for tunable far-infrared photodetectors [33].

V. CONCLUSION

In summary, we have investigated the dynamic Stark shift of a high-lying atom in a system of BEC of photons within the framework of nonrelativistic QED theory. In this configuration, the effective mass and the chemical potential for weakly interacting photons inside the microcavity are nonvanishing. Via Bogoliubov transform, sound waves exists for long-wavelength disturbances of the system. It is found that compared to that in two-dimensional free space, Stark shift of the atom in BEC of photons is modified by a factor F_{BEC}. It is a monotonically increasing function of temperature T and depends on a few other parameters of the system. Below the critical temperature T_c, the value of Stark shift is always greater than that in a normal two-dimensional photonic fluid to an order of $4 \sim 5$. Physical origin of this phenomenon is discussed and future applications is proposed including Doppler shift compensation and tunable far-infrared photodetectors. It is hoped that the predicted properties will be verified in physics laboratories for the not too distant future.

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[1] A.Einstein, Sirzber, Kgl, Preuss, Akad, Wisee **261** (1924).
[2] M. Anderson, J. Ensher, M. Matthews, C. Wieman, E. Cornell, *et al.*, Science **269**, 198 (1995).
[3] K. Davis, M. Mewes, M. Andrews, N. Van Druten, D. Durfee, D. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
[4] H. Stoof, M. Houbiers, C. Sackett, and R. Hulet, Phys. Rev. Lett. **76**, 10 (1996).
[5] T. Weber, J. Herbig, M. Mark, H. Nägerl, and R. Grimm, Science **299**, 232 (2003).
[6] G. Modugno, G. Ferrari, G. Roati, R. Brecha, A. Simoni, and M. Inguscio, Science **294**, 1320 (2001).
[7] D. Fried, T. Killian, L. Willmann, D. Landhuis, S. Moss, D. Kleppner, and T. Greytak, Phys. Rev. Lett. **81**, 3811 (1998).
[8] A. Amo, J. Lefrère, S. Pigeon, C. Adrados, C. Ciuti, I. Carusotto, R. Houdré, E. Giacobino, and A. Bramati, Nature Phys. **5**, 805 (2009).
[9] J. Klaers, J. Schmitt, F. Vewinger, and M. Weitz, Nature **468**, 545 (2010).
[10] R. Matloob, Phys. Rev. A **61**, 062103 (2000).
[11] M. Schaden, L. Spruch, and F. Zhou, Phys. Rev. A **57**, 1108 (1998).
[12] S. John and J. Wang, Phys. Rev. Lett. **64**, 2418 (1990).
[13] S.-Y. Zhu, Y. Yang, H. Chen, H. Zheng, and M. S. Zubairy, Phys. Rev. Lett. **84**, 2136 (2000).
[14] M. Chaichian, M. M. Sheikh-Jabbari, and A. Tureanu, Phys. Rev. Lett. **86**, 2716 (2001).
[15] X.-H. Wang, Y. S. Kivshar, and B.-Y. Gu, Phys. Rev. Lett. **93**, 073901 (2004).
[16] M. Yin and Z. Cheng, Phys. Rev. A **79**, 043803 (2009).
[17] M. Yin and Z. Cheng, J. Phys. B: At. Mol. Opt. Phys. **42**, 165502 (2009).
[18] L. Pitaevskii and S. Stringari, *Bose-Einstein Condensation*, Vol. 116 (Oxford University Press, USA, 2003).
[19] R. Y. Chiao, T. H. Hansson, J. M. Leinaas, and S. Viefers, Phys. Rev. A **69**, 063816 (2004).
[20] H. Yokoyama and S. Brorson, J. App. Phys. **66**, 4801 (1989).
[21] E. De Angelis, F. De Martini, and P. Mataloni, J. Opt. B **2**, 149 (2000).
[22] I. H. Deutsch and J. C. Garrison, Phys. Rev. A **43**, 2498 (1991).
[23] R. Chiao, Opt. Commum. **179**, 157 (2000).
[24] Z. Hadzibabic and J. Dalibard, arXiv preprint arXiv:0912.1490 (2009).
[25] N. Bogoliubov, J. Phys (U.S.S.R) **11** (1947).
[26] C. Kittel and H. Kroemer, *Thermal Physics* (WH Freeman, 1980).
[27] W. Mullin, J. Low Temp. Phys. **106**, 615 (1997).
[28] L. Hollberg and J. L. Hall, Phys. Rev. Lett. **53**, 230 (1984).
[29] M. L. Zimmerman, M. G. Littman, M. M. Kash, and D. Kleppner, Phys. Rev. A **20**, 2251 (1979).
[30] H. Bethe, Phys. Rev. **72**, 339 (1947).
[31] J. Sakurai, *Advanced Quantum Mechanics* (World Publication Company (Beijing), Pearson Education, 2010).
[32] J. Farley and W. Wing, Phys. Rev. A **23**, 2397 (1981).
[33] Y. Huang and C. Lien, J. Appl. Phys. **77**, 3433 (1995).